Control by social influence: durables vs. non-durables

Bary Pradelski

Control by social influence: durables vs. non-durables

Bary S. R. Pradelski*

Abstract

Individual behavior such as the adoption of new products is influenced by taking account of others' actions. We study social influence in a heterogeneous population and analyze the behavior of the dynamic processes. We distinguish between two information regimes: (i) *agents are influenced by the adoption ratio,* (ii) *agents are influenced by the usage history. We identify the stable equilibria and long-run frequencies of the dynamics. We then show that the two processes generate qualitatively different dynamics, leaving characteristic 'footprints'. In particular,* (ii) *favors more extreme outcomes than* (i)*. This has direct implications for the control of policy interventions.*

1. Introduction

A fundamental question about aggregate behavior of groups is how shifts between seemingly stable states occur almost instantaneously after long lags of low fluctuation. Social influence has been found to play an important role in such transitions. Social influence describes the process in which individuals are influenced by the behaviors of others in a group.

This paper studies two general families of dynamic processes of social influence dependent on the nature of information available. Agents update their actions at random points in time. Their decisions are influenced by the others' previous actions previously. In some environments an agent chooses a *durable* action and adoption statistics are observable or public information. In such cases an agent essentially considers the adoption ratio. We denote this scenario by *Adoption Ratio*. In other environments, each agent chooses a *non-durable* action and the act of choosing is the critical information. Hence an agent considers the usage history. We shall thus denote this scenario *Usage History*.[1]

Some environments are better described by one model and others by the other. Consider the use of bicycles for the daily commute to work. People have different personal reasons to use a bicycle or another means of transport, e.g., distance to work, personal fitness, or income. In addition, people are influenced by the behaviors of others and have a propensity to conform. Our study explores this latter influence in a heterogeneous population. On the one hand, the number of bicycle owners may be an important factor for an agent's decision (*Adoption Ratio*). On the other hand, the time series of choices, i.e., how often bicycles are seen to be used to commute to work may well be an equally important factor (*Usage History*). Knowing which process of social influence, *Adoption Ratio* or *Usage History*, is at work is relevant to control the behavior and design interventions. If the observed process follows *Adoption Ratio* an incentive to purchase a bicycle would be the right intervention. Such an intervention was used in London, UK, where the *Cyclescheme* allows employees to purchase bicycles tax-free. If the observed process follows *Usage History* an incentive should aim at increasing the frequency of choices. Again, such an intervention was used in London, UK, where *Boris Bikes* allows people to use bicycles for free for the first 30 minutes.

The contribution of this paper is twofold. First, we identify the equilibria of the *Adoption Ratio* and *Usage History* process and study their stability in a stochastic environment. Second, we find that the long-run behavior of these seemingly similar processes differs and elaborate on the qualitative differences. We show that each process leaves a characteristic footprint – *Usage History* favors extremer outcomes than *Adoption Ratio* – and provide an empirical test.

2. Related literature

The discussion of social influence has a long history in the social sciences (see, e.g., [Ham71, Shi00]).[2]

*Univ. Grenoble Alpes, CNRS, Inria, Grenoble INP, LIG 38000, France bary.pradelski@cnrs.fr

[1] Note that this differentiation has not gone unnoticed in marketing departments. Hardware providers such as Apple focus their reporting and marketing effort on the number of products sold (*Adoption Ratio*). Software providers, such as WhatsApp, focus their analysis and messaging on the frequency of usage of their products (*Usage History*).

[2] For further studies see [You09, CMP18].

Numerous applications have motivated the study of social influence, e.g., political and social movements [Sch78, Cab12], control of diffusion processes such as innovation adoption [Rog62, Bas69, MI06, NS15], and financial herding [Ban92, Bou13]. Consequently, there is broad experimental evidence for social influence. [Asc55] conducted a series of enlightening experiments showing that a considerable proportion of subjects trust the majority over their own senses. More recently [SDW06] show the effects of social influence in a study on music taste. Other applications studies include human fertility [BW96], diffusion of information technologies [TGG02], mobile phones [dSRZ11], and bicycle usage for the work commute [GR11].

Our model follows [Sch78] and [Gra78]. They describe the class of *critical mass models* of social interaction. [Sch78] notes that "though perhaps not in physical and chemical reactions, in social reactions it is typically the case that the 'critical number' for one person differs from another's." Thus the *tipping value* determines, for each player, the critical mass of the aggregate information about the population's actions at which a player will 'tip over' from playing one action to another. We study a threshold model where heterogeneous players repeatedly revise their binary action. Our model is in discrete time with asynchronous updating.

The processes we consider are Markovian. On the one hand, we use the concept of *stochastic stability* [FY90, KMR93, You93].[3] The idea is to study a perturbed version of the original process, such that the resulting Markov process is irreducible and ergodic and therefore the process has a unique stationary distribution. By letting the level of noise approach zero one can identify those states that will be observed in the long-run with a frequency bounded away from zero. On the other hand, we also make use of recent work on reinforced random walks [Pin13]. Pinsky analyzes a random walk on $\mathbb{Z}$ whose probability of moving left or right depends on the recent history. By an appropriate translation we find the unique limit proportion of play even though all states are stochastically stable and thus stochastic stability does not allow any selection result.

3. The model

We shall first introduce the general framework for analyzing social influence. Let $P=\{1,...,p\}$, $p\in\mathbb{N}$ be the set of players. Let $A=\{0,1\}$ be the actions available to each player $i\in P$.[4] Let $u_i:[0,1]\times A\to\mathbb{R}$ be the utility of agent $i\in P$ when observing the aggregate statistic about the society $s\in[0,1]$ and playing action $a\in A$. We shall define two specific functional forms for s later but for now it suffices to think of some aggregate statistic about the players' actions. Suppose that the utility of an action is separable into a component arising from a player's inherent preference for an action and a component specifying the utility he derives from social conformity. After normalizing, let $\pi_i\in\mathbb{R}$ be player i's direct utility difference when playing action 1 over action 0. Further let $\rho_i\in\mathbb{R}^+$ be a player's *index of social conformity*. Finally, suppose that the impact of social influence is linear. A player's utility from playing action a is then given by

$$u_i(s,a)=\begin{cases}\pi_i+\rho_i s & \text{if } a=1,\\ \rho_i(1-s) & \text{if } a=0.\end{cases} \tag{1}$$

This is a coordination game when s is increasing in the number of players playing 1. A player is indifferent between the actions when $u_i(s,1)=u_i(s,0)$. That is, agent i is indifferent iff

$$s=\frac{\rho_i-\pi_i}{2\rho_i}=:\mu(i) \tag{2}$$

We shall call $\mu(i)$ player i's *tipping value*. If $s>\mu(i)$ player i wants to play 1 and if $s<\mu(i)$ he wants to play 0. A player with $\mu(i)\in(-\infty,0)$ always prefers to play action 1 and a player with $\mu(i)\in(1,\infty)$ always prefers action 0. We shall make the simplifying assumption that for all players i, $\mu(i)$ is not a multiple of $1/p$ which will ensure that a player always has a unique best response (*BR*). Given the list of different tipping values $\mu_1,...,\mu_n$ (players may have the same tipping values, hence $n\le p$) let q_j be the fraction of players with tipping value μ_j, i.e., $q_j=\frac{\sum_{i=1}^p \mathbf{1}_{\mu(i)=\mu_j}}{p}$ (for $j=1,\dots,n$).

Let $f_i:\mathbb{R}^2\to[0,1]$ be the *response function* for player i, specifying the probability to play action 1 given his utilities $u_i(s,1)\in\mathbb{R}$ and $u_i(s,0)\in\mathbb{R}$. Note that $1-f_i(\cdot,\cdot)$ is the probability that i plays action 0. We shall initially consider a best-response model:

$$f_i=\begin{cases}1 & \text{if } s>\mu(i),\\ 0.5 & \text{if } s=\mu(i),\\ 0 & \text{else.}\end{cases} \tag{3}$$

We study a process where in each period $t=0,1,2,\dots$ a unique player gets activated. In a given period t the activated player i will be called *active*. Define

$$\mathbf{1}_i^t=\begin{cases}1 & i \text{ is active in } t,\\ 0 & \text{else.}\end{cases} \tag{4}$$

We write $act(t)\in P$ for the player who is active in period t. Let $s(t)\in[0,1]$ be the aggregate statistic about society at the beginning of period t. For each player i, let a_i^t be the action he plays at time t. Then for all $i\in P$

$$a_i^t=\mathbf{1}_i^t\cdot B^t[f_i(u_i(s(t),1),u_i(s(t),0))]+(1\text{-}\mathbf{1}_i^t)a_i^{t-1} \tag{5}$$

for all $t\ge1$, where $(B^t)_{t\in\mathbb{N}}$ is a family of independent Bernoulli RVs taking values in A. Let $\bar{a}^t=\sum_{i=1}^p a_i^t/p\in$

[3] [Pes10] extend the model of [KMR93] to general networks. Our work provides an extension to heterogeneous players as in [EK10].

[4] To avoid unnecessary notation we name actions such that their sum indicates the average action taken.

$[0, \frac{1}{p}, \ldots, 1]$ be the population's average action in period t. We consider two processes of social influence, arising from responding to different observations about society:

Adoption Ratio. The state at the beginning of period t is given by the action profile $\mathbf{a}^{t-1} = (a_i^{t-1})_{i \in P}$. An active player responds to the *Adoption Ratio*:

$$s^{AR}(t) = \bar{a}^{t-1} \tag{6}$$

Usage History. The state at the beginning of period t is given by the last k actions, i.e., $(a_{act(t-k)}^{t-k}, \ldots, a_{act(t-1)}^{t-1})$. An active player responds to the *Usage History* in the past k (constant) periods:[5,6]

$$s^{UH}(t) = \frac{\sum_{v=t-k}^{t-1} a_{act(v)}^{v}}{k} \tag{7}$$

When unambiguous we shall sometimes omit the specification of the time period. To illustrate, suppose there are four players. Player 1 initially plays 1 and all other players play 0. Suppose we are in time step $t = 7$ and play unfolded as shown in table 1. For *Adoption Ratio* the observation in period $t = 7$ of prior adopters of action 1 is $s^{AR}(7) = 50\%$ (the relevant actions are circled in Table 1). For *Usage History* (with $k = 5$) the observation in period $t = 7$ of the frequency of choices of action 1 is $s^{UH}(7) = 80\%$ (the relevant actions are boxed in Table 1). Note that in our example player 3's previous actions have no influence on the observed time series while player 1 and 2's are counted twice.

Table 1. Actions up to $t = 6$

time, t	0	1	2	3	4	5	6
active player	–	2	1	4	1	2	2
player 1	1		$\boxed{1}$		$\boxed{1}$		
player 2	0	1				$\boxed{1}$	$\boxed{1}$
player 3	0						
player 4	0			$\boxed{0}$			

Relevant actions for $s^{AR}(7)$ circled, relevant actions for $s^{UH}(7)$ boxed ($k = 5$).

We analyze the two processes for several regimes of sampling and errors. Initially, we study the unperturbed best-response dynamic. We then consider a uniform action tremble. That is, with small probability $\varepsilon > 0$ an activated player picks an action uniformly at random. The response function for player i then is

$$f_i = \begin{cases} 1 - \frac{\varepsilon}{2} & \text{if } s > \mu(i), \\ 0.5 & \text{if } s = \mu(i), \\ \frac{\varepsilon}{2} & \text{else.} \end{cases} \tag{8}$$

[5]For *Usage History*, we need to define $s(t)$ differently for $t < k$. We can simply assume the average of the past t actions.

[6]Note that $s(t)$ includes the active player's action. This is reasonable when players are presented with the aggregate statistic, but the analysis also carries through if one excludes the active player's action from $s(t)$.

4. Analysis

We shall first state a definition and a simple lemma. All the proofs can be found in the full paper on the author's webpage (www.barypradelski.com).

Definition 1. *Given the population's average action $\bar{a}$, let Agg give the share of players for whom 1 is a best-response when observing $\bar{a} \in [0,1]$:*

$$Agg : [0,1] \to \left\{0, \frac{1}{p}, \frac{2}{p}, \ldots, 1\right\} \tag{9}$$

$$Agg(\bar{a}) = \frac{1}{p} \sum_{i=1}^{p} \mathbf{1}_{\mu(i) < \bar{a}} \tag{10}$$

Lemma 2. *Agg has at least one fixed point. If x^* is a fixed point of Agg all players of the same type have the same BR in the associated state and hence play the same action. Formally, let $\bar{q}_k := \sum_{j=1}^{k} q_j$. Then*

$$x^* \in \{\bar{q}_k : \mu_k < \bar{q}_k < \mu_{k+1} \text{ for some } k \in \{1, \ldots, n\}\}.^7 \tag{11}$$

We denote by $\mathbf{a}_\alpha^* \in \{\mathbf{a}_1^*, \ldots, \mathbf{a}_l^*\}$ the action profiles where all players play a *BR* and $\bar{a}(\mathbf{a}_\alpha^*) = x_\alpha^*$ and $x_\alpha^* \in \{x_1^*, \ldots, x_l^*\}$ is a fixed point of *Agg* (in increasing order).

4.1. Adoption Ratio

In this section we consider the social influence process *Adoption Ratio*. An active player bases his decision on the number of current adopters (see Eq. (6)).

$$s^{AR}(t) = \bar{a}^{t-1} = \frac{\sum_{i \in P} a_i^{t-1}}{p}$$

Theorem 3. *The unperturbed* Adoption Ratio *dynamic has at least one absorbing state. The absorbing states of the dynamic process coincide with the fixed points of Agg and each absorbing state is associated with exactly one fixed point of Agg and vice versa. The set of absorbing states which can be reached is dependent on the initial state (when multiple absorbing states exist).*

4.1.1. Perturbed dynamics. We now consider the perturbed process with uniform error (see Eq. 8).

Theorem 4. *Suppose players have* uniform action trembles *in the* Adoption Ratio *dynamic. The stochastically stable states are those states in $\boldsymbol{a}_\alpha^* \in \{\boldsymbol{a}_1^*, \ldots, \boldsymbol{a}_l^*\}$ which are associated with $x_\alpha^* \in \{x_1^*, \ldots, x_l^*\}$ of Agg that minimize*

$$\boldsymbol{a}_\alpha^* : \gamma_{\boldsymbol{a}_\alpha^*} = \min_{\beta = 1, \ldots, l} \gamma_{\boldsymbol{a}_\beta^*} \tag{12}$$

[7]Set $\mu_{n+1} = 1$ for completeness.

where $\gamma_{\boldsymbol{a}^*_\alpha}$ is the stochastic potential:

$$\gamma_{\boldsymbol{a}^*_\alpha} = \sum_{\beta=1}^{\alpha-1} r_{x^*_\beta, x^*_{\beta+1}} + \sum_{\beta=\alpha+1}^{l} r_{x^*_\beta, x^*_{\beta-1}}, \text{ with} \tag{13}$$

$$r_{x^*_\beta, x^*_{\beta+1}} = \max_{x \in \left[x^*_\beta, x^*_{\beta+1}\right]} \{x - Agg(x)\} \cdot p + 1 \tag{14}$$

$$r_{x^*_{\beta+1}, x^*_\beta} = \max_{x \in \left[x^*_\beta, x^*_{\beta+1}\right]} \{Agg(x) - x\} \cdot p + 1 \tag{15}$$

For generic games there exists a unique stable state.

The former two equations describe the maximal number of trembles (i.e., non-*BR*s) needed to exit the basin of attraction of a given fixpoint x^*_β and to enter the basin of attraction of the neighboring fixpoint $x^*_{\beta+1}$.

4.2. Usage History

In this section we shall consider the social influence process *Usage History*. An active player bases his decision on the time series of choices (see Eq. (7)):

$$s^{UH}(t) = \frac{\sum_{v=t-k}^{t-1} a^v_{act(v)}}{k}$$

Theorem 5. *The unperturbed* Usage History *dynamic has absorbing states iff* 0 *and/or* 1 *are fixed points of Agg or the BR of all players is independent of social influence (i.e., for all players $i \in P$ the tipping value $\mu(i) \notin [0,1]$). In the former case all $-$ 0 and/or all $-$ 1 are the unique absorbing states. In the latter case the unique absorbing state is the unique fixed point of Agg.*

4.2.1. Perturbed dynamics.

We now consider the perturbed process with a uniform error (see Eq. 8). Recall that q_j is the fraction of players with tipping value μ_j.

Theorem 6. *Suppose players have* uniform action trembles *in the* Usage History *dynamic.*

If the unperturbed dynamic has an absorbing state 0 *(*1*) is stochastically stable if* $r_{0,1} \geq r_{1,0}$ *(*$r_{1,0} \geq r_{0,1}$*).*

Else, let $\bar{q}_k = \sum_{j=1}^{k} q_j$. For an action profile $\boldsymbol{a}^*_\alpha \in \{\boldsymbol{a}^*_1, \ldots, \boldsymbol{a}^*_l\}$ associated with fixed point $x^*_\alpha \in \{x^*_1, \ldots, x^*_l\}$ (in increasing order), let

$$\underset{j \in \{1,\ldots,n\}: \mu_j < \bar{q}_j < \mu_{j+1}}{\arg\max} \frac{1}{\bar{q}_j^{\mu_j}(1-\bar{q}_j)^{1-\mu_j}} \cdot \prod_{k=2}^{j} \left(\frac{\bar{q}_{k-1}}{1-\bar{q}_{k-1}}\right)^{\mu_k - \mu_{k-1}} = \{j_1, \ldots, j_\Delta\} \tag{16}$$

Then the limit proportion of play of action 1 *is given by*

$$\lim_{k\to\infty} \lim_{t\to\infty} \lim_{\varepsilon\to 0} \frac{\sum_{v=0}^{t} a^v_{act(v)}}{t} = \frac{\sum_{\delta=1}^{\Delta} \frac{1}{1-\bar{q}_{j_\delta}} \cdot q_{j_\delta}}{\sum_{\delta=1}^{\Delta} \frac{1}{1-\bar{q}_{j_\delta}}} \tag{17}$$

For generic games there exists a unique maximizer of Eq. (16)*, say j_*. The latter formula then reduces to:*

$$\lim_{k\to\infty} \lim_{t\to\infty} \lim_{\varepsilon\to 0} \frac{\sum_{v=0}^{t} a^v_{act(v)}}{t} = q_{j_*} \tag{18}$$

5. Adoption Ratio vs. Usage History: Example

We shall now formalize the example from the introduction. Consider bicycle usage and suppose that some commuters use the bicycle irrespective of its popularity, say innovators. Further, there is an early and late majority who may use the bicycle if enough others use it. Finally, there are some non-adopters who will never use a bicycle for their daily commute. In particular assume the following population shares and thresholds:

5% *innovators*, always us the bicycle for their commute to work (action 1), i.e., they play the innovation independent of social influence and hence their tipping value is 'negative' ($\mu_{\text{innovators}} < 0$),

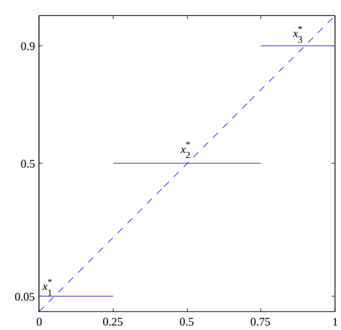
45% *early majority*, who use the bicycle if at least 'few' use it (e.g., $\mu_{\text{early majority}} = 25\%$),

40% *late majority*, who use the bicycle if at least 'many' use it (e.g., $\mu_{\text{late majority}} = 75\%$),

10% *non-adopters*, who never play the innovation ($\mu_{\text{non-adopters}} > 1$).

Figure 1 shows the function *Agg* and the fixed points x^*_1, x^*_2, x^*_3. We invite the reader to verify that the fixed points of *Agg* are $x^*_1 = 5\%$, $x^*_2 = 50\%$, $x^*_3 = 90\%$.

Figure 1. x-axis shows $\mu(i)$, y-axis shows Agg



We compute the long-run stable state under *Adoption Ratio* according to Theorem 4. One finds the following resistances[8] $r_{x^*_1,x^*_2} = 0.2$, $r_{x^*_2,x^*_3} = 0.25$, $r_{x^*_2,x^*_1} = 0.25$, $r_{x^*_3,x^*_2} = 0.15$ and the stochastic potentials $\gamma_{x^*_1} = 0.4$, $\gamma_{x^*_2} = 0.35$, $\gamma_{x^*_3} = 0.45$. Thus the (unique) stochastically stable state is x^*_2 since it uniquely minimizes stochastic potential. Next, we compute the long-run stable state under *Usage History* according to Theorem 6. The rounded results of Eq. (16) are: $x^*_1 : 1.05$, $x^*_2 : 0.96$, $x^*_3 : 0.92$. The long-run frequency of action 1 is given by x^*_1. This shows, by example, that the two different processes *Adoption Ratio* and *Usage History* may

[8] Note that to be precise we need to define the population size and add resistance one to each of the formulas below. But this does not change the result.

yield significantly different outcomes. Note that the example is generic in the following sense: We can define "close-by" distributions with the same outcome. To build intuition consider fixed point $x_1^* = 5\%$. To reach the basin of attraction of x_2^* the time series of choices (over the last k periods) needs to be at least 25%. That is, in the last k periods, players for whom 1 is currently the *BR* (5% of the population) need to be activated at least $25\% \cdot k$ of the time. On average such a player needs to be activated at least 5 times as often as players whose *BR* is currently 0. On the other hand, suppose we are currently in $x_2^* = 50\%$. To reach the basin of attraction of x_1^* the time series of choices (over the last k periods) needs to be at most 25%. That is, in the last k periods, players for whom 0 is currently the *BR* (50% of the population) need to be activated at least $(100\% - 25\%) \cdot k$ times. That is, on average such a player needs to be activated at least 1.5 times as often as players whose *BR* is currently 1.[9] Since 1.5 is less than 5 it follows that the latter transition is more likely than the former.

6. Empirically discriminating between *Adoption Ratio* and *Usage History*

We here show characteristic footprints in the distribution of s^{AR} and s^{UH} for the perturbed dynamics that can be used to empirically discriminate which process is at work (as long as $x^* = 0.5$ is currently not the observed fixed point and s^{UH} has no absorbing states in the unperturbed process). It turns out that s^{AR} is not binomially distributed but s^{UH} is. In addition, the variance of the two processes are, in general, different. This enables us to empirically discriminate which process is at work and thus to inform policy interventions or marketing campaigns. We analyze the behavior of s^{AR} and s^{UH} around an interior fixed point x^* of the dynamic. We shall study the distribution of s^{AR} and s^{UH} conditional on remaining in the basin of attraction of x^*.

Distribution of s^{AR}. Suppose that $\varepsilon > 0$ is fixed. Note that within a basin of attraction a player's *BR* remains constant. Then a player's action, when activated to revise, is picked independently of the other players' actions. Given his response function f_i, he plays his *BR* with probability $1 - \frac{\varepsilon}{2}$ and the other action with probability $\frac{\varepsilon}{2}$ (Bernoulli trial). That is, $p \cdot x^*$ players play d with probability $f_i = 1 - \varepsilon/2$ and m otherwise, and $p \cdot (1 - x^*)$ players play d with probability $f_i = \varepsilon/2$ and m otherwise. We thus have for s^{AR}:

$$
\begin{aligned}
s^{AR} &\sim \frac{1}{p}\left(\textstyle\sum_{i \in P: f_i = 1-\varepsilon/2} B(1, 1 - \frac{\varepsilon}{2}) + \sum_{i \in P: f_i = \varepsilon/2} B(1, \frac{\varepsilon}{2})\right) && (19)\\
&\sim \frac{1}{p}\left(B(p \cdot x^*, 1 - \tfrac{\varepsilon}{2}) + B(p \cdot (1 - x^*), \tfrac{\varepsilon}{2})\right) && (20)
\end{aligned}
$$

where Eq. (20) holds since the sum of iid binomials is again binomial with the same parameter. By [BS93] there is no closed form for the distribution of a sum of binomials with different parameters, but they derive a recursive formula. For the reduced form in Eq. (19) of two binomials one easily finds for the mean of s^{AR}:

$$
\begin{aligned}
\mathbb{E}(s^{AR}) &= x^* \cdot (1 - \frac{\varepsilon}{2}) + (1 - x^*) \cdot \frac{\varepsilon}{2} && (21)\\
&= x^* + (1 - 2x^*) \cdot \frac{\varepsilon}{2} && (22)
\end{aligned}
$$

and for the variance of s^{AR}:

$$
\begin{aligned}
Var(s^{AR}) &= x^* \cdot (1 - \frac{\varepsilon}{2}) \cdot \frac{\varepsilon}{2} + (1 - x^*) \cdot \frac{\varepsilon}{2} \cdot (1 - \frac{\varepsilon}{2}) && (23)\\
&= \frac{\varepsilon}{2} \cdot (1 - \frac{\varepsilon}{2}) && (24)
\end{aligned}
$$

Distribution of s^{UH}. As before, within a basin of attraction a player's *BR* remains constant. Since we consider the case where the unperturbed process has no absorbing states we can assume that $\varepsilon = 0$. Then the next action (by the player activated in the next period) is given by a binomial distribution with parameter x^* (since players best respond with probability 1). The intuition, that s^{UH} follows the sum of k binomial distributions with parameter x^* is wrong, since the order of occurrence matters and thus $s^{UH}(t)$ is correlated with $s^{UH}(t+i)$ for all $i = 1, \ldots, k-1$. By considering non-overlapping windows of length k we can recover independence and thus have that for $j = 1, 2, 3, \ldots$ $s^{UH}(k \cdot j)$ are independent Bernoulli trials with parameter x^*. Thus $s^{UH}(k \cdot t)$ is binomial distributed with parameter x^* and one finds for the mean of s^{UH}:

$$\mathbb{E}[s^{UH}(k \cdot t)] = x^* \qquad (25)$$

and for the variance of s^{UH}:

$$Var(s^{UH}(k \cdot t)) = x^* \cdot (1 - x^*) \qquad (26)$$

To summarize s^{AR} is not binomially distributed (for $x^* \neq 0.5$) and s^{UH} is binomially distributed. Further the variances of the two processes are, in general, different. Thus we can employ standard statistical tests to identify which process, *Adoption Ratio* or *Usage History*, is underlying a given sample of the aggregate observation data. Note that it is not necessary to have any additional information, in particular it is not necessary to know the players' thresholds or order of activation. The first test we can use is whether the observed data is binomially distributed. If this yields a statistically significant result we are done. But if ε is very small it may be that the result of this test is not statistically significant. Then a second test can be used. The variance of *Usage History* depends on the fixed point (i.e., the mean) and we can use this fact to discriminate between the two processes. In particular, if the fixed point is not too close to $all - m$ or $all - d$ we can use this test. Finally, if data of the

[9] This follows from the simple calculation $(100\% - 25\%)/50\% = 1.5$.

behavior around two fixed points is available a third test can be performed. For *Adoption Ratio* the variance is the same around all fixed points whereas the variance for *Usage History* differs around each fixed point.

7. Conclusion

In this paper we have studied the dynamics of social influence. We considered two different processes of social influence. On the one hand, social influence arises from the *Adoption Ratio* of the agents' actions. On the other hand, social influence arises from the *Usage History* of choices. We first identified the equilibria of the two processes and studied their stability in stochastic environments. We then showed that the outcomes may be very different. Thus one needs to carefully examine the specific process of social influence at hand in order to be able to control outcomes and design interventions. Returning to our example on bicycle usage discussed in the introduction the knowledge of which process is at hand may inform whether an intervention to promote the purchase of a bicycle (e.g., Cyclescheme) or an intervention to promote the usage of bicycles (e.g., Boris Bikes) is more apt.

8. ACKNOWLEDGMENTS

I thank D. Helbing, M. Maes, H. Nax, J. Ruf, B. Sandholm, P. Tarrès, H. P. Young, and various seminar participants for helpful comments.

References

[Asc55] S. E. Asch. Opinions and social pressure. *Scientific American*, 193(5):31–35, 1955.

[Ban92] A. V. Banerjee. A simple model of herd behavior. *Quarterly Journal of Economics*, 107(3):797–817, 1992.

[Bas69] F. M. Bass. A new product growth model for consumer durables. *Management Science*, 15:215–227, 1969.

[Bou13] J.-P. Bouchaud. Crises and collective socio-economic phenomena. cartoon models and challenges. *Journal of Statistical Physics*, 151(3-4):567–606, 2013.

[BS93] K. Butler and M. Stephens. The distribution of a sum of binomial random variables, 1993. Technical Report 467, Stanford University, April 1993, prepared for the Office of Naval Research.

[BW96] J. Bongaarts and S. C. Watkins. Social interactions and contemporary fertility transitions. *Population and Development Review*, 22(4):639–682, 1996.

[Cab12] Cabinet Office. *Applying behavioural insights to reduce fraud, error and debt*. Behavioural Insights Team UK, 2012.

[CMP18] B. Clemm von Hohenberg, M. Maes, and B. S. R. Pradelski. Micro influence and macro dynamics of opinion formation, 2018. Working Paper.

[dSRZ11] H. de Silva, D. Ratnadiwakara, and A. Zainudeen. Social influence in mobile phone adoption: Evidence from the bottom of the pyramid in emerging asia. *Information Technologies & International Development*, 7(3):1–18, 2011.

[EK10] D. Easley and J. Kleinberg. *Networks, crowds, and markets: Reasoning about a highly connected world*. Cambridge University Press, 2010.

[FY90] D. Foster and H. P. Young. Stochastic evolutionary game dynamics. *Theoretical Population Biology*, 38:219–232, 1990.

[GR11] F. Goetzke and T. Rave. Bicycle use in Germany: Explaining differences between municipalities with social network effects. *Urban Studies*, 48(2):427–437, 2011.

[Gra78] M. Granovetter. Threshold models of collective behavior. *The American Journal of Sociology*, 83(6):1420–1443, 1978.

[Ham71] W. D. Hamilton. Geometry for the selfish herd. *Journal of Theoretical Biology*, 31:295–311, 1971.

[KMR93] M. Kandori, G. J. Mailath, and R. Rob. Learning, mutation, and long run equilibria in games. *Econometrica*, 61(1):29–56, 1993.

[MI06] N. Meade and T. Islam. Modelling and forecasting the diffusion of innovation - a 25-year review. *International Journal of Forecasting*, 22:519–545, 2006.

[NS15] J. Newton and R. Sawa. A one-shot deviation principle for stability in matching problems. *Journal of Economic Theory*, 157:1–27, 2015.

[Pes10] M. Peski. Generalized risk-dominance and asymmetric dynamics. *Journal of Economic Theory*, 145:216–248, 2010.

[Pin13] R. G. Pinsky. The speed of a random walk excited by its recent history, 2013. Working Paper, arXiv: 1305.7242.

[Rog62] E. M. Rogers. *Diffusion of Innovations*. The Free Press, 1962.

[Sch78] T. C. Schelling. *Micromotives and Macrobehavior*. Norton & Company, 1978.

[SDW06] M. J. Salganik, P. S. Dodds, and D. J. Watts. Experimental study of inequality and unpredictability in an artificial cultural market. *Science*, 311(5762):854–856, 2006.

[Shi00] R. J. Shiller. *Irrational exuberance*. Princeton University Press, 2000.

[TGG02] J. T. C. Teng, V. Grover, and W. Guettler. Information technology innovations: General diffusion patterns and its relationships to innovation characteristics. *IEEE Transactions on Engineering Management*, 49(1):13–27, 2002.

[You93] H. P. Young. The evolution of conventions. *Econometrica*, 61(1):57–84, 1993.

[You09] H. P. Young. Learning by trial and error. *Games and Economic Behavior*, 65:626–643, 2009.